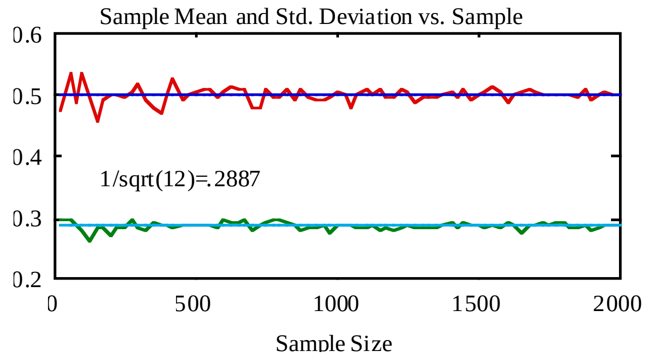
Search and Detection - HW1

1. Using MATLAB, generate 25 iid U[0,1] random variables. Compute the sample mean and sample standard deviation. Repeat for sample sizes of 50, 75, 100, ... , 2000. Plot sample mean and sample standard deviation vs. sample size. Your final plot should look something like the one on the right. Notice how the variation decreases as sample size increases.



2. In MATLAB, generate a row vector x of 1000 iid U[0,1] random variables as x = RAND(1,1000).

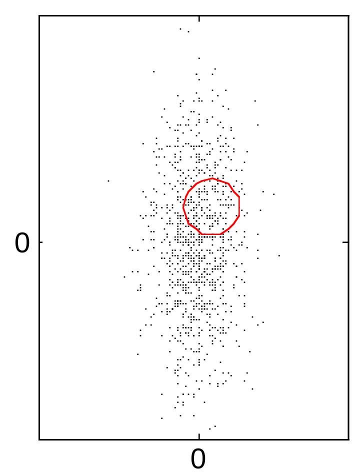
a. For this random sample, calculate the sample mean as MEAN(x) and sample standard deviation as STD(x).
b. Estimate the standard deviation of the sample mean as STD(x)/SQRT(1000).
c. Estimate the 95% confidence interval for the population mean as MEAN(x)±1.96*STD(x)/SQRT(1000).

Note: The estimated confidence interval in 2.c. is based on (1) the Central Limit Theorem which requires that for sufficiently large sample sizes (say, 30 or more), the sample mean is approximately normally distributed around the population mean; (2) approximately 95% of the probability mass of any normal random variable is within 1.96 standard deviations of the mean; and (3) the population standard deviation is approximated by the sample standard deviation, again for sufficiently large samples.

d. Calculate 100 confidence intervals as in 2.c. How many actually contain the correct value of .5?

3. The impact point of a mortar round is bivariate normal (BVN) with means (0,0) and standard deviations (5,15). That is, the y- or downrange position is normally distributed with mean 0 and standard deviation 15; and the x- or crossrange position is an independent normal random variable with mean 0 and standard deviation 5. Distances are in meters. A target is at position (3,8), and this target will be damaged if a round lands at or within a distance of 6 meters.

a. Use MATLAB to generate 1000 sample impact points. Construct the 1000-element vector d, where d(i)=1 if round i is on the damage disk and d(i)=0 otherwise. Estimate the 1-round probability of damage (Pd) as MEAN(d) (≈.15) and the standard deviation of this estimate as STD(d)/SQRT(1000) (≈.01).



Note: For any iid Bernoulli (i.e., binary) random sample b of size n,
STD(b)/SQRT(n) = SQRT(MEAN(b)*(1-MEAN(b))/(n-1)).
If MEAN(b) has already been computed, the RHS is more computer-efficient than is the LHS to estimate the standard deviation of the sample mean. But the RHS expression can not be used in Problem 2.c. because the random variables there are not binary.

c. Estimate the 95% confidence interval for the probability of damage as MEAN(d)±1.96*STD(d)/SQRT(1000).

d. Verify numerically that STD(d)/SQRT(1000) = SQRT(MEAN(d)*(1-MEAN(d))/999). Use the MATLAB statement FORMAT LONG to compute your answers to 15 places.